

#### Multiplication and division

#### EYFS:

Children will learn to solve problems, including doubling, halving and sharing.

EYFS end of year expectations	Concrete	Pictorial	Abstract	Using and applying
Solves problems, including doubling, halving and sharing (ELG). (Numbers)	Using objects to show double.  Using objects to show half and to share.	Drawing pictures to show double and to half or 'share'.	Doubles  1 + 1  2 + 2  3 + 3  4 + 4	If Megan has 3 toys and Maheen has 3 toys, how many toys do they have altogether?

Multiplication and division (Note: 'units' are now called 'ones')

#### Key Stage 1:

- The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources (for example, concrete objects and measuring tools).
- By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

End of year expectations	Rapid recall	Mental calculation	Language	Using and applying
Year 1		Count on and back in 2, 5	Groups of	Solve simple one-step problems that involve using
		and 10.	Array	concrete objects and pictorial representations.
			Counting in	
			Sharing	
			Double	
			Half	
			Quarter	

#### Multiplication and division

Year 2	Identifying odd and even	Count in steps of 3 from 0	Odd, even,	Solve one-step problems involving multiplication
	numbers.	and in tens from any	Repeated	and division, using materials, arrays, repeated
	Recall multiples of 2, 5 and	number, forward or	addition/subtraction	addition, mental methods, and multiplication and
	10 and related division facts.	backward.	Grouping/ sharing	division facts, including problems in contexts.
			Inverse	
			Multiply Multiple(s) of	They connect the 10 multiplication table to place
			Divide	value, and the 5 multiplication table to the
			Division	divisions on the clock face.
			Commutative	
			Calculate	
			Equivalent	

#### Multiplication and division

Year 1	Concrete	Pictorial	Conceptual	Using & applying
Multiplication and division as repeated addition and subtraction	Using familiar objects and resources.  Finding 'groups of' with repeated addition and subtraction.	Repeated images E.g. How many legs?	2+2+2 5+5+5+5	Making links If one teddy has two apples, how many apples will three teddies have?
Represent repeated addition as an array.  Begin to use arrays to find repeated subtraction.	Make arrays on grids with counting objects	Understand visual representations of arrays	2+2+2 5+5+5+5 12-3-3-3-3=0	Here are 10 lego people If 2 people fit into the train carriage, how many carriages do we need?  Practical If we put two pencils in each pencil pot how many pencils will we need?

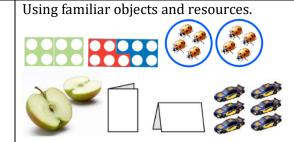
#### Multiplication and division

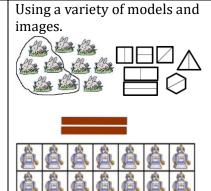
(Note: 'units' are now called 'ones')

Doubling and halving numbers within 20 (as repeated addition and subtraction).

Voor 2

multiplication





Using number sentences and beginning to calculate mentally.

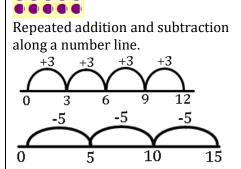
6 + 6 =Double 9 = 14 = Double ...
Half of 18 = ...  $\frac{1}{2}$  of  $\square = 5$  10 = half of....  $7 = 14 - \square$   $4 + \square = 8$ 

Class 1 has 8 girls.
Class 2 has **double** the number of girls.
How many girls are there in Class 2?

rear Z	Concrete
Use arrays to	Make arrays on grids using
make or draw	counting objects.
multiplications and find the corresponding division facts.	
Calculate mathematical statements for multiplication and division	
within the	Identify arrays in everyday

objects.

Concrete



Pictorial
Array images

Using number sentences and beginning to calculate mentally.

 $3 \times 4 = 12$  $12 \div 4 = 3$ 

Missing number problems.



 $20 = \Box x 5$ 

3 = □ ÷ 6

Using and applying
I had 20 lollies.
I put them into groups of 5.
How many groups were there?
I had 20 lollies.
I shared them between 5 people.
How many lollies did

I saved 5p **each** week for 6 weeks.

each person get?

#### Multiplication and division

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tables and write them using the	000			How much did I save altogether?
multiplication (×), division (÷) and equals (=) sign				If I save 5p <b>each</b> week, how many weeks will it take me to save 40p?
Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.	Make arrays on grids using counting objects. $2 \times 4 = 8$ Rotating arrays to find other multiplications. $4 \times 2 = 8$	Repeated addition and subtraction along a number line. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Using number sentences and beginning to calculate mentally. $3 \times 5 = 15$ $5 \times 3 = 15$ $15 \div 5 = 3$ $15 \div 3 = 5$	There are 24 parents coming to watch our class assembly. How many different ways can you arrange the chairs? (In equal rows).

Multiplication and division (Note: 'units' are now called 'ones')

#### **Lower Key Stage 2:**

- The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.
- At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

End of year expectations	Rapid recall	Mental calculation	Language	Using and applying
Year 3	Recall multiples of 2, 5 and 10 and related division facts.  Begin to recall multiples of 3, 6 and 4 and 8 and related division facts.	Count from 0 in multiples of 4, 8, 50 and 100.	Grid method Product Short division Remainder	Pupils should solve simple problems in contexts, including missing number problems, deciding which of the four operations to use and why, including measuring and scaling contexts, and correspondence problems in which m objects are connected to n objects (e.g. 3 hats and 4 coats, how many different outfits; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

#### Multiplication and division

	Recall multiplication and division facts for multiplication tables up to 12 × 12	Count in multiples of 6, 7, 9, 25 and 1 000  Multiply 3 numbers 0 x 0 x 0  Recall factor pairs for a given number  Multiply by 0 and 1 and divide by 1	Factor Factor pair Quotient Divisor	choosing the appro increasingly harder	e <b>two-step</b> problems in contexts, opriate operation, working with r numbers. This should include lestions such as three cakes shared 0 children.
Year 3	Concrete	Pictorial	Conceptua	al	Using & applying
TO x O  Extending understanding of arrays (TO x O), progressing to formal written methods	Tens onics	Use arrays to link to grimultiplications.  Tens Units  X 20 3  4 80 12  80 + 12 = 92  Children can also write expanded calculations	id Using num $18 \times 4 = \square$ $31 \times 3 = \square$ Missing nu $\square \times 41 = 12$ $7 \times \square = 84$	ber sentences. mber problems.	Year 3 went on a trip. There were 6 groups with 14 children in each group. How many children went on the trip in total?  Use the digits 2, 3, 4, 5 and 6.  Make a multiplication (0 x T0) e.g. 2 x 53 = Find different totals can you find?  How many multiplications have the <i>same</i> total?

Multiplication and division

 _		7	
Tens	Units	the grid e.g. 4x20=80 and	
10 10	1 1 1	4x3=12.	
10 10	1 1 1		
10 10	1 1 1		
10 10	1 1		
Begin to link to operations: 92 ÷ 4 = 23	to inverse		

#### Multiplication and division

Note: units are now called ones j							
Year 3	Concrete	Pictorial	Conceptual	Using & applying			
TO ÷ 0  Sharing and grouping to create an	Using counting objects and resources.	Repeated subtraction on a number line.	Short division.  23 3 69	69 children were <b>grouped</b> equally onto 3 buses for a trip. How many children went on each bus?			
array.  (Not exchanging from tens to units at this stage).	$69 \div 3 = 23$ $\begin{array}{cccccccccccccccccccccccccccccccccccc$	Extending divisions to resemble written method of short division.  23  3  9  9  1  1  1  1  1  1  1  1  1  1  1	Check using multiplication inverse: $ \begin{array}{c cccc} x & 20 & 3 \\ \hline 3 & 60 & 9 \\ \hline 60 + 9 = 69 \end{array} $	3 children <b>shared</b> £69 equally. How much did they each receive?  How many different divisions can you make? 36 ÷? =?			
Understand the concept of remainders after division.	Using resources. 23 ÷ 4 = 5 r3	Repeated addition and subtraction along a number line. $23 \div 4 = 5 \text{ r}3$ $ \begin{array}{cccccccccccccccccccccccccccccccccc$	Begin to solve mentally. $23 \div 4 = \square$ $31 \div 6 = \square$ Missing number problems. $\square \div 3 = 4^{r1}$ $17 \div \square = 3^{r2}$	A farmer had 33 eggs. He put them into boxes of 6. How many <b>full</b> boxes did he have? How many eggs did he have left over? If he put them into boxes of 12, how many would be left over now?			

#### Multiplication and division

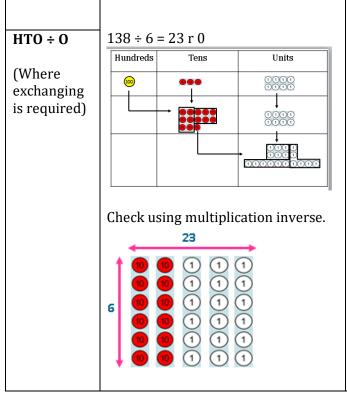
(Note: um	its are now caned ones j			
				Use each number in the 4x table. Make it with counters then share it into 3 groups. Write the remainder each time. What patterns do you notice?
Year 4	Concrete	Pictorial	Conceptual	Using and applying
multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout	Crossing <i>one</i> boundary.  126 x 3 =  Hundreds Tens Units	Beginning with grid multiplication.	Expanded method (if children need this) $126$ $\times 3$ $18 (3 \times 6)$ $60 (3 \times 20)$ $300 (3 \times 100)$ $378$ If children are ready, move onto compact vertical method. $1 2 6$ $\times 3$ $3 7 8$ $1$	In one week, 163 people visited the museum each day. How many people visited in total?  My sister and I were raising money for charity. We collected £127 every day for 6 days. We <b>shared</b> the money <b>equally</b> between two different charities. How much money did each charity receive?
	Extending to crossing <i>two</i> boundaries.	Beginning with grid multiplication.	Expanded method (if children need this)	Use the digits 1, 2, 3 and 5. Make a multiplication U x

#### Multiplication and division

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	247 x 3 =    Hundreds   Tens   Units	600 + 120 + 21	$ \begin{array}{r} 247 \\ x     3 \\ 21     (3 x 7) \\ 120     (3 x 40) \\ \underline{600}     (3 x 200) \\ \underline{741} \end{array} $ If children are ready, move onto compact vertical method. $ \begin{array}{r} 2     4     7 \\ x     3 \\ \underline{7     4     1} \\ 1     2 \end{array} $	HTU. How many different products are there? What are the largest and smallest products possible?  O x HTO = 820. How many ways can you solve this?
Year 4	Concrete	Pictorial	Conceptual	Using and applying
T0 ÷ 0  (Where exchanging is required)	Grouping and sharing using place value counters. Exchanging counters which cannot be grouped.	Result of grouping/sharing counters during 'concrete' stage.		A school ordered 432 pencils. They were put into packs of 5. How many packs were made? How many pencils were left over?

#### Multiplication and division

(Note: 'units' are now called 'ones')

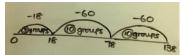


Chunking on a number line.

$$138 \div 6 = 23$$

Key Facts 1 x 6 = 6 2 x 6 = 12 5 x 6 = 30

 $10 \times 6 = 60$ 



Short division methods.

 $138 \div 6 = 23$ 

432 ÷ 5 becomes

Answer: 86 remainder 2

Robbie has 150 stickers. He kept 12 and shared the rest equally between 6 friends. How many stickers did each of his friends get?

436 children need to be put into teams for sports day. How many different ways could the children be grouped equally?

How many divisions can you make which have a remainder of 3? What patterns do you notice?

Which numbers between 100 and 150 have a remainder of 1 when they are divided by 2, 3, 4, 5, and 6?

Multiplication and division (Note: 'units' are now called 'ones')

#### <u>Upper Key Stage 2:</u>

- The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.
- At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems.
- By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

End of year expectations	Rapid recall	Mental calculation	Language	Using and applying
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#### Multiplication and division

Year 5	Related decimal facts for tables E.g. 6 x 7 = 42 0.6 x 7 = 0.7 x 6 = 4.2÷7= etc.	Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000.	Prime number Composite number Common factors Square / cube numbers	Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
Year 6		Perform mental calculations, including with mixed operations and large numbers  E.g. 3 x 700 + 115 =  Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.		Use their knowledge of the order of operations to carry out calculations involving the four operations.  Solve addition and subtraction multi-step. Problems in contexts, deciding which operations and methods to use and why.  Solve problems involving addition, subtraction, multiplication and division including interpreting remainders appropriately within the context of the problem.

Multiplication and division

dentify common factors, common nultiples and prime numbers	
sse their knowledge of the order of operations to carry out calculations involving the four operations	

Year 5	Concrete	Pictorial	Conceptual	Using and applying
Multiply	Refer to Year 4 (HTO x 0) and	Use grid method	Refer to Year 4 expanded	There are 5 kittens, each
numbers	extend the process of using place	HTO x O and extend to	vertical method (HTO x O)	weighing 1352g. What is
up to <b>four</b>	value counters to ThHTO x O	ThHTO x O	and extend to ThHTO x O	their total mass in Kg?
digits by a				
1 or 2-	(Year 4)	(Year 4)		Use the digits 1 to 5. Make
digit	Crossing <i>one</i> boundary.	Beginning with grid multiplication.	126	a multiplication: ThHTO x
number	126 x 3 =	x 100 20 6	<u>x 3</u>	O. How many products can
using a			18 (3 x 6)	you make between 5000
formal	Hundreds Tens Units	3  300   60   18	60 (3 x 20)	and 5500?
written	(ii)	300 + 60 + 18	<u>300</u> (3 x 100)	
method,		300 1 00 1 10	<u>378</u>	
including				
long	0			
multiplica			Short multiplication.	
tion for 2-				

#### Multiplication and division

	ints are now caned ones	)		T
digit numbers	NP: Children should proceed to	Crid mathada	2 7 4 1 <u>X</u> 6 1 6 4 4 6 4 2 Answer: 2741 x 6= 16446	Leaved £26 every week for
ТОхТО	NB: Children should proceed to pictorial methods alongside methods used in year 4.	Grid method: $47 \times 36 =                                 $	Compact method:  47	I saved £36 every week for a year. At the end of the year, I gave half of it to charity. How much money did I donate?  Try this with several numbers: choose a prime number greater than 3, square it and divide the answer by 12. Look at the remainder. What do you notice? Why does this happen?
НТО x ТО	Follow processes shown above (TO x TO).	Grid method.  382 x 23 =    x   300   80   2     2   20 x   20 x 80   20 x 2 =     0   300 =   =   40	Compact method:  1 2 4  x 26  7 4 4  2 4 8 0  3 2 2 4	There are 24 bottles in a crate. Each bottle has a capacity of 720ml. what is the total amount in litres?

## Multiplication and division (Note: 'units' are now called 'ones

(Note: 'ur	nits' are now called 'ones'				
		6000 1600 3 3 x 3 x 80 = 240 900 240  6000 40 900 240 + 6 8786 1	3 x 2 = 6	(20 x 124) Answer: 124 x 26 = 3224	Make 5 different 2 digit numbers e.g. 56, 74, 31, 65, 83. Multiply them each by 101. What do you notice? What happens when you multiply each one by 1001?
ThHTO ÷ O divide numbers up to four digits by a 1-digit number using the formal written method of short division and	Follow processes shown in Year 4 HTO ÷ 0 with place value counters	Chunking on a number  Children should use the ke help them with related fact $10 \times 7 = 70$ $100 \times 7 = 700$ $1256 \div 7 =$	y facts box to	Short division $432 \div 5 \text{ becomes}$ $\begin{array}{c ccccccccccccccccccccccccccccccccccc$	6 people won £8724 on the lottery. They spent £650 on a party to celebrate then shared the rest. How much did they each receive?  How many divisions can you create which leave a remainder of 4/5, 2/3etc.?

#### Multiplication and division

interpret	The are now caned ones	)	0 F 0 n 2	
remainder s appropriat ely for the context		Answer: 179 remainder 3 or 179 <u>3</u> 7	$ \begin{array}{c} 858 \text{ r } 2 \\ 3 \overline{\smash)2^2 5^1 7^2 6} \\ \text{Answer: } 858 \frac{2}{3} \end{array} $	Try this with several numbers: choose a prime number greater than 3, square it and divide the answer by 12. Look at the remainder. What do you notice? Why does this happen?
Year 6	Concrete	Pictorial	Conceptual	Using and applying
Multiply	NB: Children should proceed to	Use grid method for ThHTO xTO (refer to	Compact method	There are 24 bottles in a
multi-digit	pictorial methods .	year 5). Grid can also be used to multiply decimal	1735	crate. Each bottle has a
numbers up		numbers.	x 43	capacity of 720ml. what is
to 4 digits by a two-		Use the grid method of multiplication (as	5 2 0 5	the total amount in litres?
digit whole		below) Grid method	211	
number		372 x 24 is approximately 400 x 20 = 8000	69400	Make 5 different 2 digit
using the		Extend to decimals with up to two decimal	212	numbers e.g. 56, 74, 31, 65,
efficient		places. 3.42 x 6 =	<u>74605</u>	83. Multiply them each by
written		x 3 0.4 0.02	1 1735 x 43 = 74605	101. What do you notice?
method of		A 0 0.7 0.02	1/35 x 43 = /4005	What happens

#### Multiplication and division

long		6	18	2.4	0.12			
multiplicati	-     -	18.0	)					
on.		2.4						
		0.1	12					
		20.5	52					

#### Multiplication and division

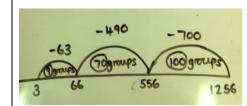
(Note: 'units' are now called 'ones')

(Note. ui	11(
Divide	
numbers	
up to four	
digits by a	
2-digit	
whole	
number	
using the	
formal	
written	
method of	
long	
division,	
and	
interpret	
remainder	
s as whole	
number	
remainder	
S,	
fractions,	
or by	
rounding,	
as	
appropriat	
e for the	
context	

Chunking on a number line.

Key Facts
$1 \times 7 = 7$
$2 \times 7 = 14$
5 x 7 = 35
10 x 7 = 70

Children should use the key facts box to help them with related facts.



Answer: 179 remainder 3 or 179  $\frac{3}{2}$ 

Short division



496 ÷ 11 becomes

Answer:  $45\frac{1}{11}$ 

Both methods above are necessary at this stage, to deal with the wide range of problems experienced at Stage Six.

432 ÷ 15 becomes

Answer:  $28\frac{4}{5}$ 

There are 432 guests at a wedding. Each table at dinner seats 15 people. How many tables are needed?

A farmer had 450 eggs. 18 smashed so he put the rest into boxes of 15. How many boxes did he use?

How many divisions can you create which result in a recurring decimal? Can you find a pattern in the numbers you used?

Choose a 4 digit number and investigate fractional and decimal remainders when you divide by 9. What patterns do you notice?

Multiplica (Note: 'ur	ation and division nits' are now called 'ones'	)	

Multiplication and division (Note: 'units' are now called 'ones')